

Geometry I, 2006 : Coursework 3

Show your calculations

1. [unassessed] Do exercises 1 and 2 on page 24 of the course text, and check your answers in the back of the book.

2. Let $\underline{u}$ and $\underline{v}$ be any vectors.

(i) [1 mark] State Theorem 2 from your lecture notes, which gives a formula for the scalar product of two vectors which are given in co-ordinates. [You may make use of this theorem in the remainder of the coursework.]

(ii) [2 mark] Prove that $\underline{u} \cdot \underline{v} = \underline{v} \cdot \underline{u}$.

(iii) [2 marks] Prove that $\underline{u} \cdot (\alpha \underline{v}) = \alpha(\underline{u} \cdot \underline{v}) = (\alpha \underline{u}) \cdot \underline{v}$ for all scalars α .

(iv) [2 marks] Apply (iii) to prove that $\underline{u} \cdot (-\underline{v}) = -(\underline{u} \cdot \underline{v}) = (-\underline{u}) \cdot \underline{v}$.

3. Let $\underline{p} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$.

(i) [1 mark] Determine the cosine of the angle between $\underline{p}$ and $\underline{q}$.

(ii) [1 mark] Determine (in co-ordinates) the vector $((3\underline{p} + 2\underline{q}) \cdot (5\underline{p} - 3\underline{q}))(\underline{p} + \underline{q})$.

4. (i) [1 mark] Find a Cartesian equation for the plane through the point $(-1, 4, 5)$ and orthogonal to the vector $\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$.

(ii) [1 mark] Is the point $(5, 0, 1)$ on this plane? Why or why not?

(iii) [1 mark] Determine the distance of the point $(1, -2, 2)$ from this plane.

5. Let $P = (1, 2, -3)$, $Q = (1, 0, 1)$, $R = (-2, 1, 2)$.

(i) [1 mark] Determine (in co-ordinates) the position vectors $\underline{p}, \underline{q}, \underline{r}$ of the points P, Q, R .

(ii) [2 marks] Determine (in co-ordinates) the vector $\underline{a}$ represented by $\overrightarrow{PQ}$ and the vector $\underline{b}$ represented by $\overrightarrow{PR}$.

(iii) [1 mark] Is $\underline{a}$ parallel to $\underline{b}$? Why or why not? [The definition of *parallel to* was given in the previous coursework.]

(iv) [2 marks] By trial and error, or any method you wish, determine (in co-ordinates) a non-zero vector $\underline{n}$ orthogonal to both $\underline{a}$ and $\underline{b}$.

(v) [2 marks] Determine a Cartesian equation for the plane containing P, Q, R .

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 19th October.

Leonard Soicher, 12 October 2006